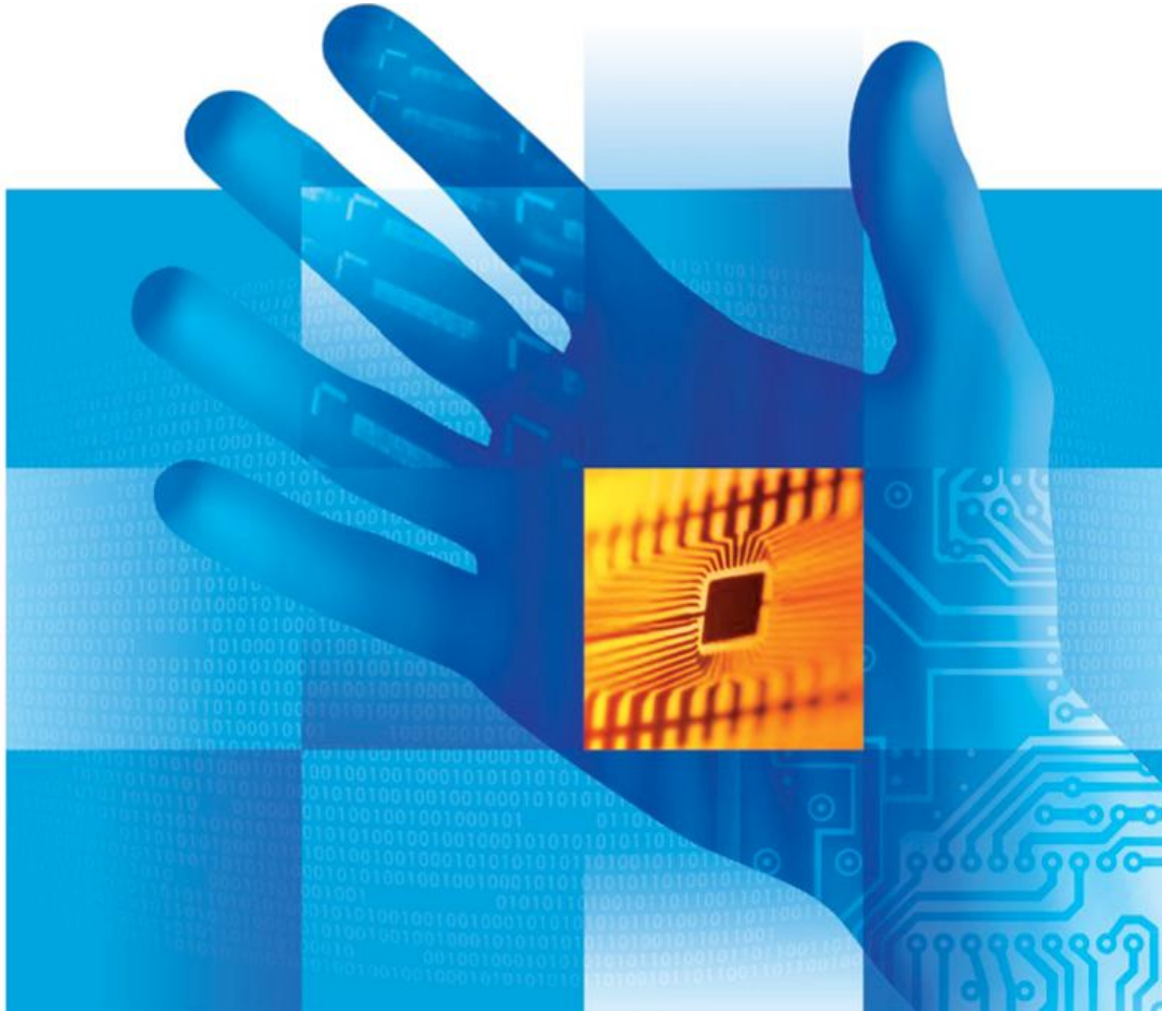
EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

3. 3-PHASE ELECTRICAL CIRCUITS, POWER FACTOR



Doc. Ing. Stanislav Kocman, Ph.D.

2. 11. 2017, Ostrava

LECTURE PLAN

- origin and advantages of three-phase system
- star and delta connections
- non-symmetrical three-phase system
- power factor correction, methods and benefits

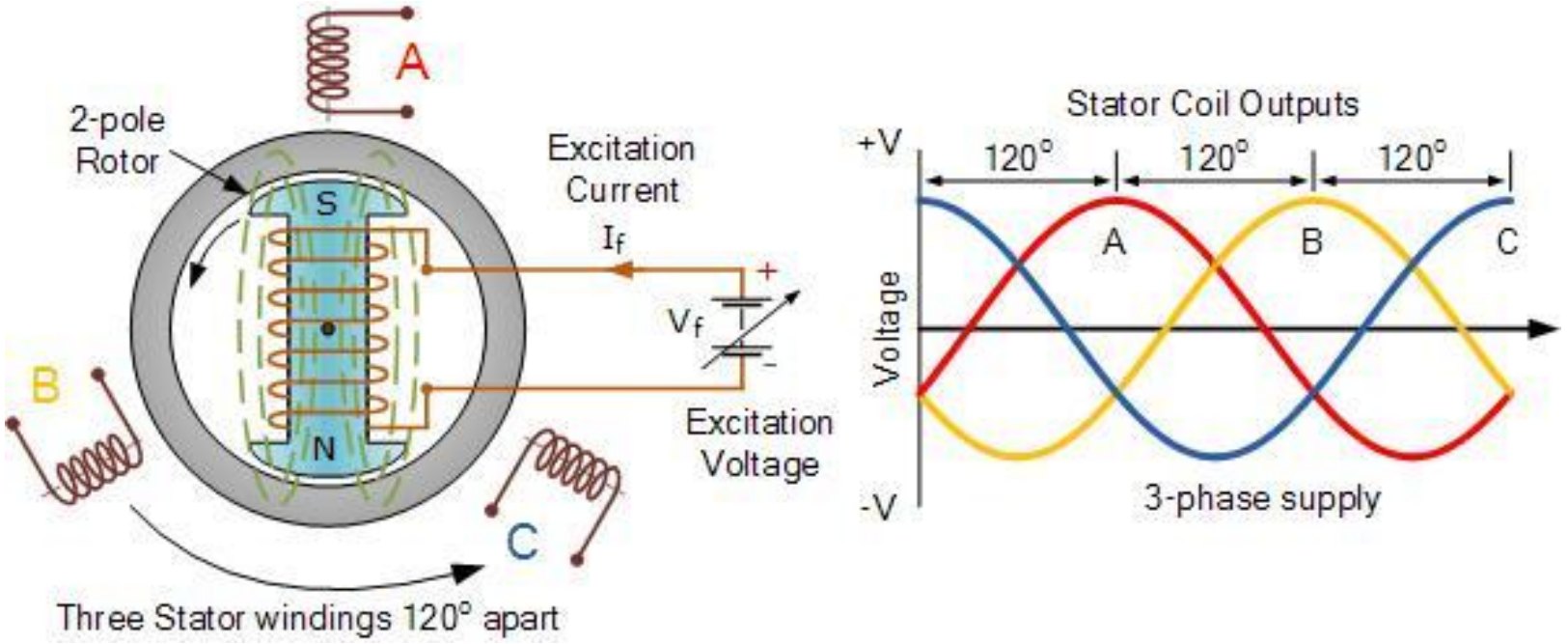
Origin and advantages of three-phase system

Advantages of the 3-phase system

- ❑ creates rotational magnetic field
- ❑ higher power and economy of generators, transformers and motors
- ❑ more economic electric power transmission in our country
- ❑ constant three-phase power in time

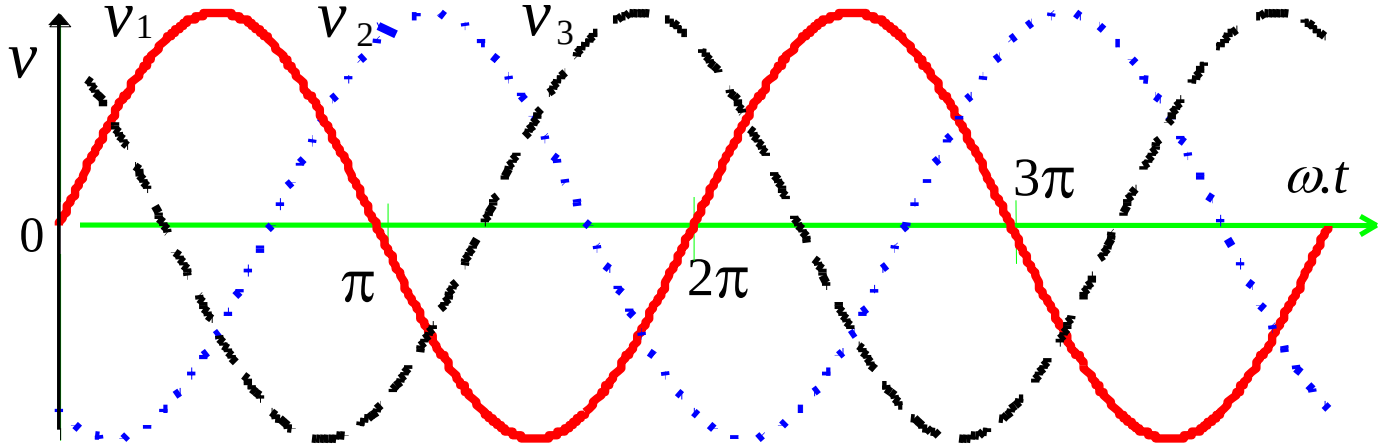
Origin of the 3-phase system

Simplified model of 3-phase generator as a source of the three-phase voltage



Symmetrical 3-phase system

Voltage waveforms



$$v_1 = V_m \cdot \sin(\omega t)$$

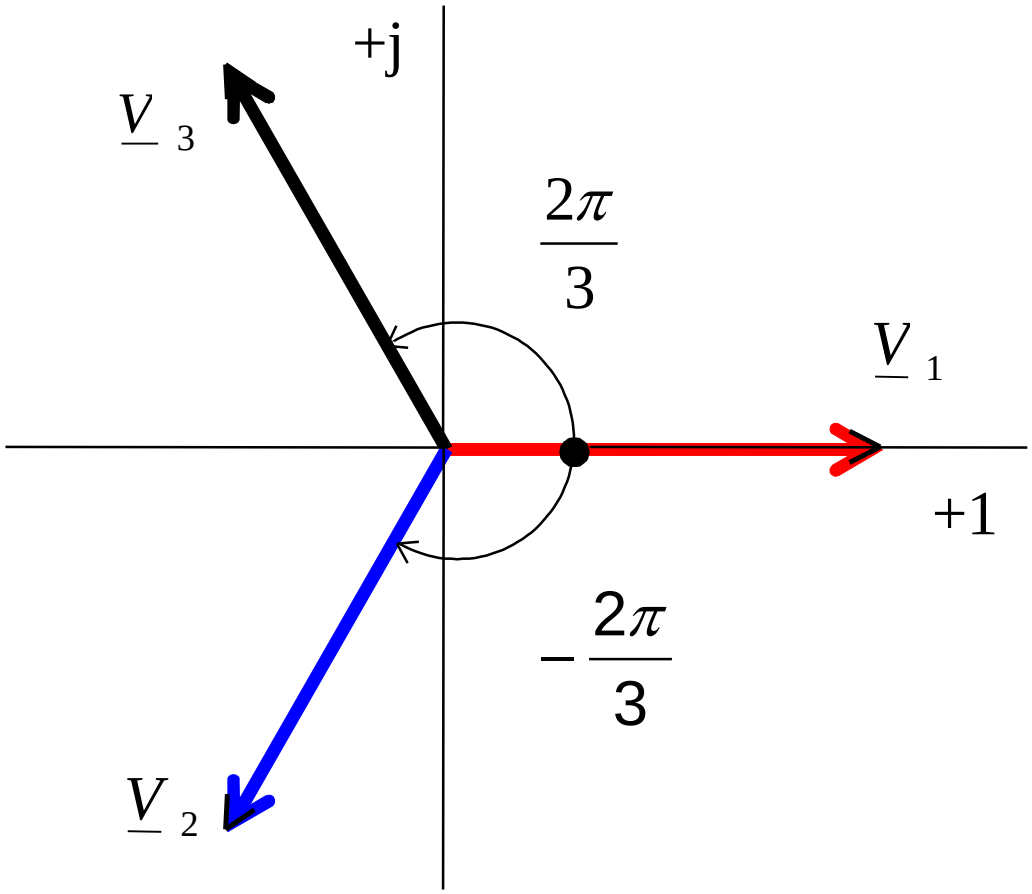
$$v_2 = V_m \cdot \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_3 = V_m \cdot \sin\left(\omega t - \frac{4\pi}{3}\right) = V_m \cdot \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$v_1 + v_2 + v_3 = 0$$

⇒ **symmetrical system**

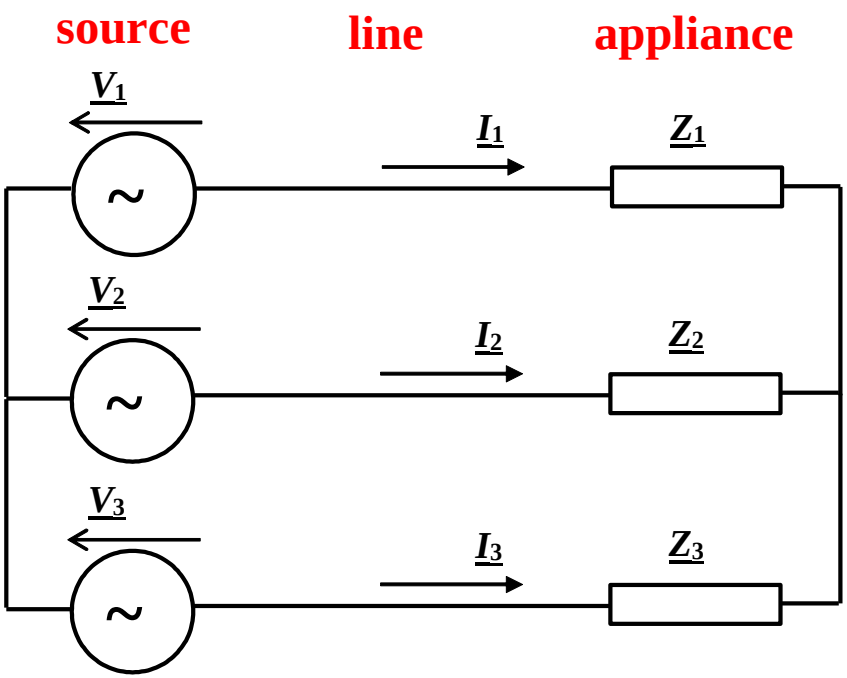
Phasor diagram



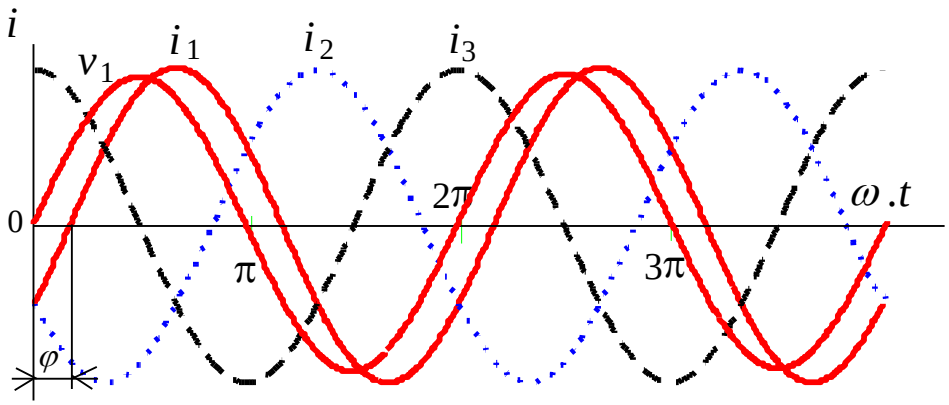



SIEMENS

Symmetrical three-phase circuit

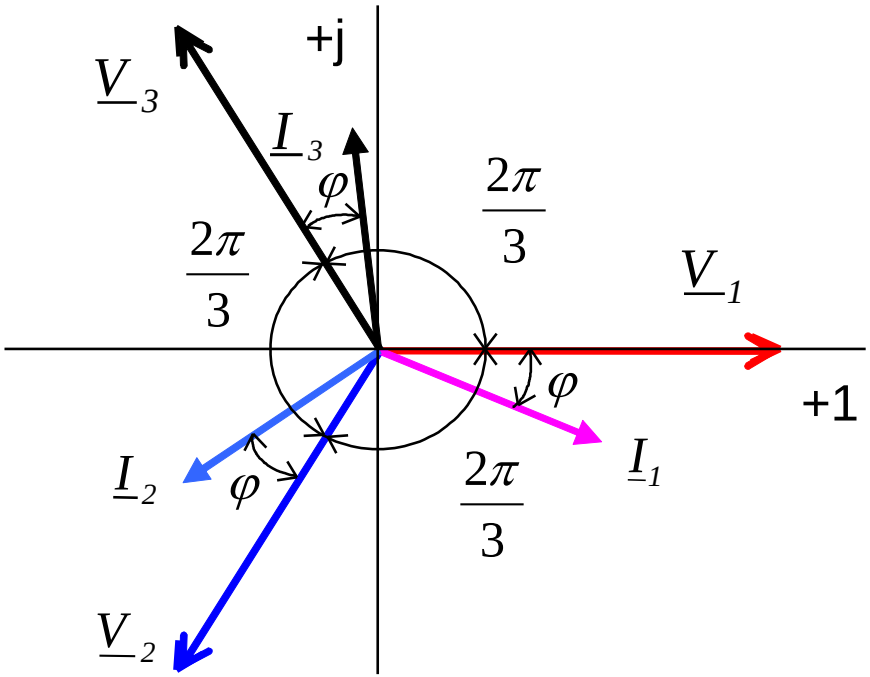


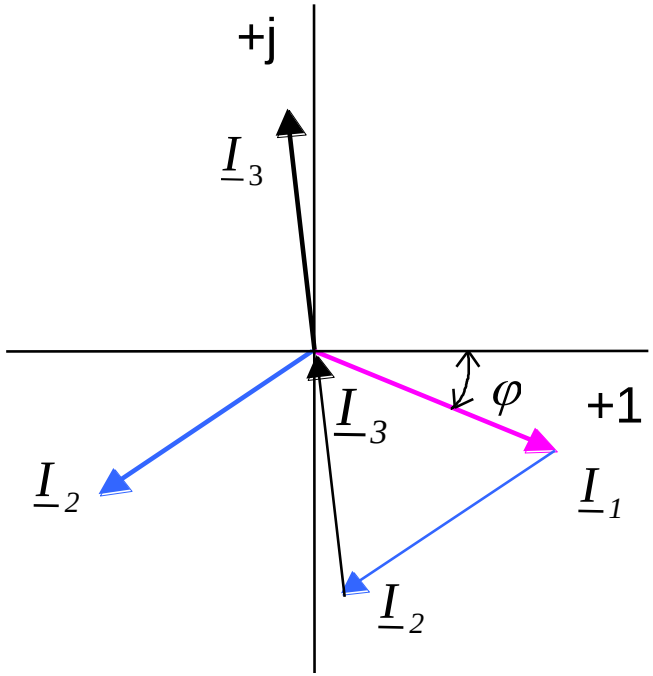
$$\underline{Z_1} = \underline{Z_2} = \underline{Z_3} \quad \left(Z_1 = Z_2 = Z_3 = Z, \varphi_1 = \varphi_2 = \varphi_3 = \varphi\right)$$



$$\boldsymbol{i}_1 + \boldsymbol{i}_2 + \boldsymbol{i}_3 = \mathrm{O}$$

Phasor diagram





$$I_1 = I_2 = I_3 = I$$

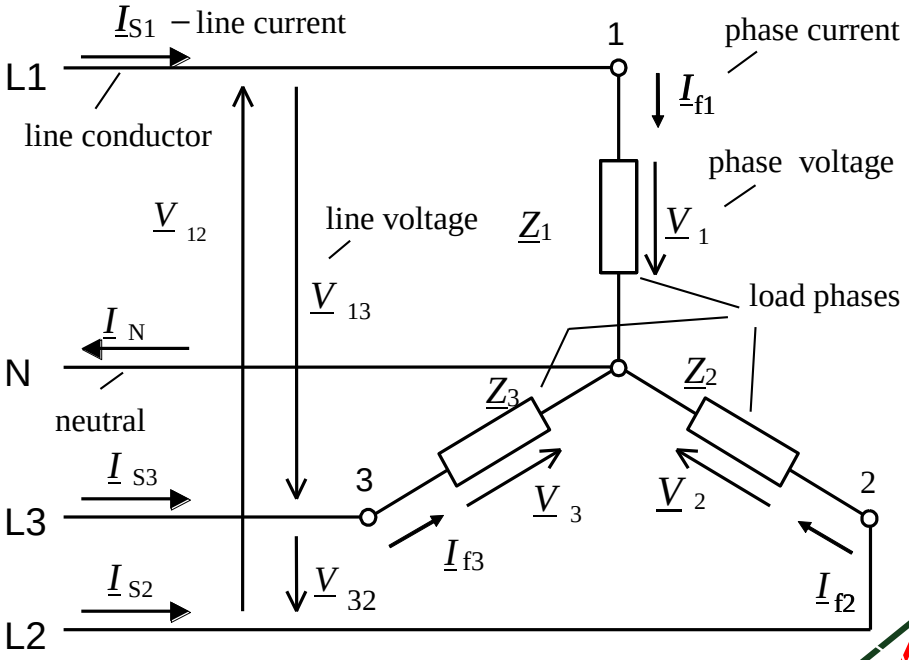
$$I = \frac{V}{Z}$$

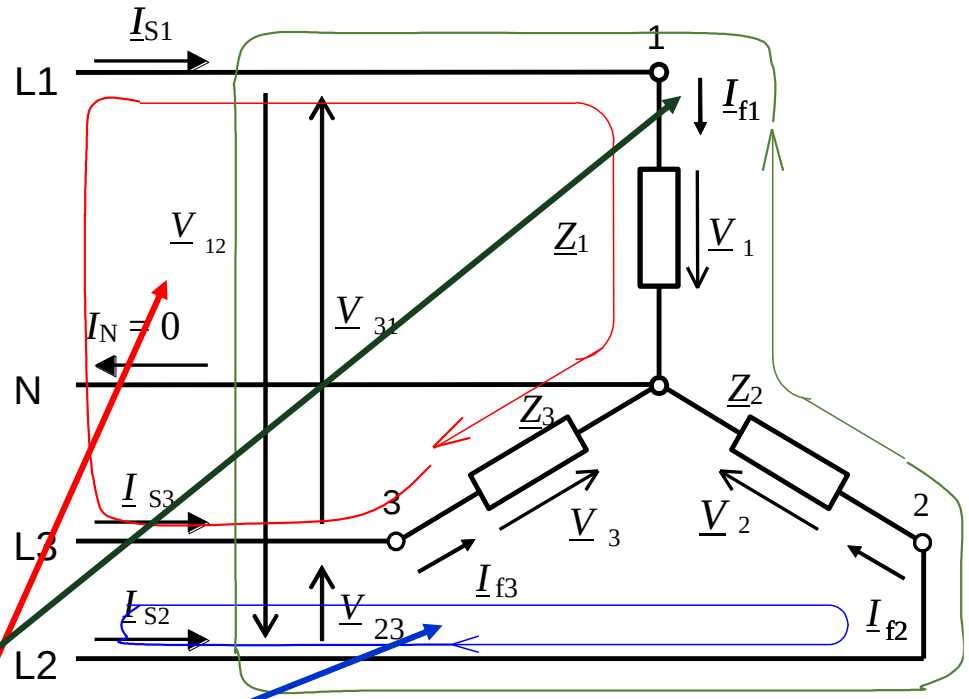
$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

Star and delta connections

Star connection

symmetrical system



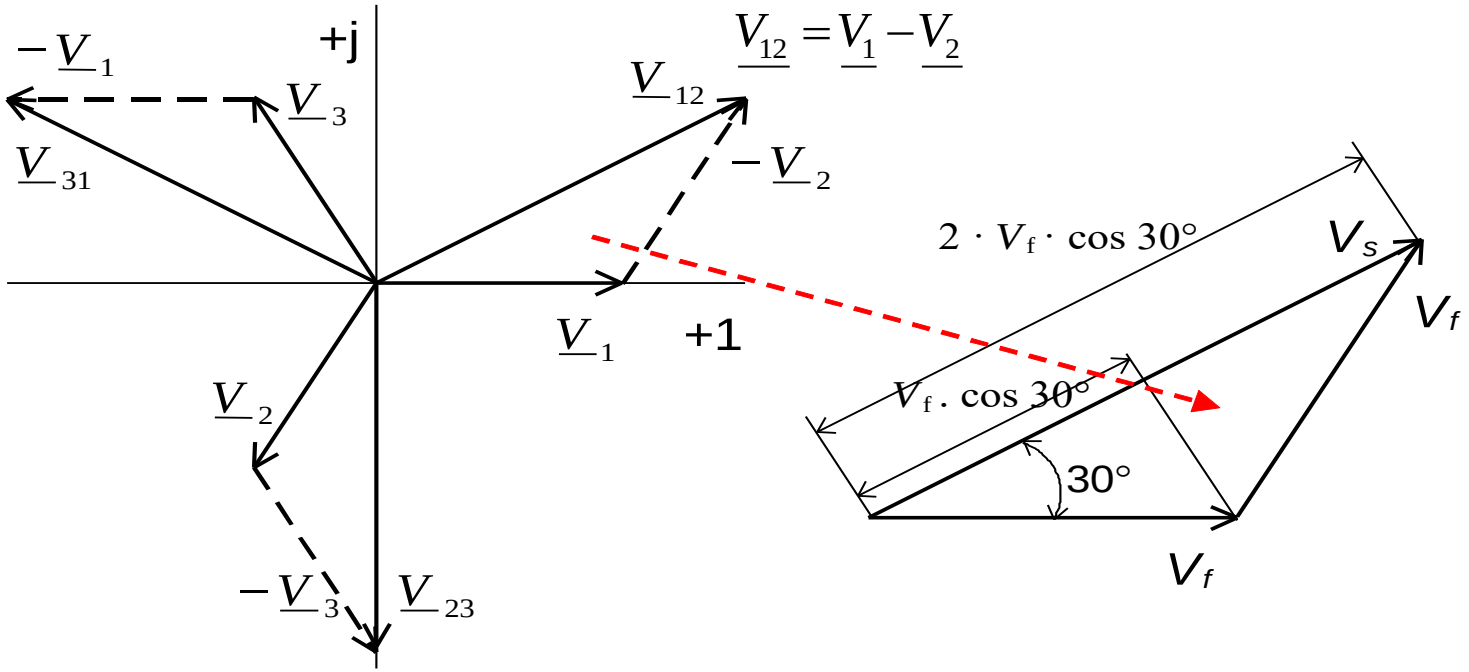


$$\underline{V}_{12} + \underline{V}_2 - \underline{V}_1 = 0 \rightarrow \underline{V}_{12} = \underline{V}_1 - \underline{V}_2$$

$$\underline{V}_{23} + \underline{V}_3 - \underline{V}_2 = 0 \rightarrow \underline{V}_{23} = \underline{V}_2 - \underline{V}_3$$

$$\underline{V}_{31} + \underline{V}_1 - \underline{V}_3 = 0 \rightarrow \underline{V}_{31} = \underline{V}_3 - \underline{V}_1$$

Voltages and currents in the star connection



$$V_1 = V_2 = V_3 = V_f$$

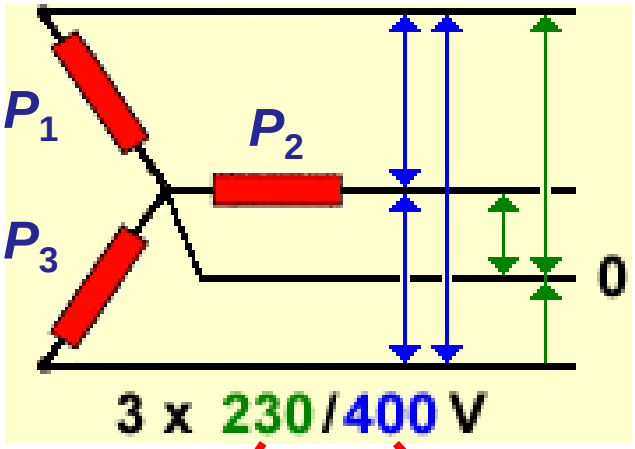
$$V_{12} = V_{23} = V_{31} = V_S$$

$$V_S = 2 \cdot V_f \cdot \cos 30^0 \rightarrow V_S = \sqrt{3} \cdot V_f$$

$$I_{f1} = I_{f2} = I_{f3} = I_f \ , \ I_f = I_S \qquad I_N = 0$$

Power in the star connection

real power
in one phase



phase voltage line voltage

real power of three phase load

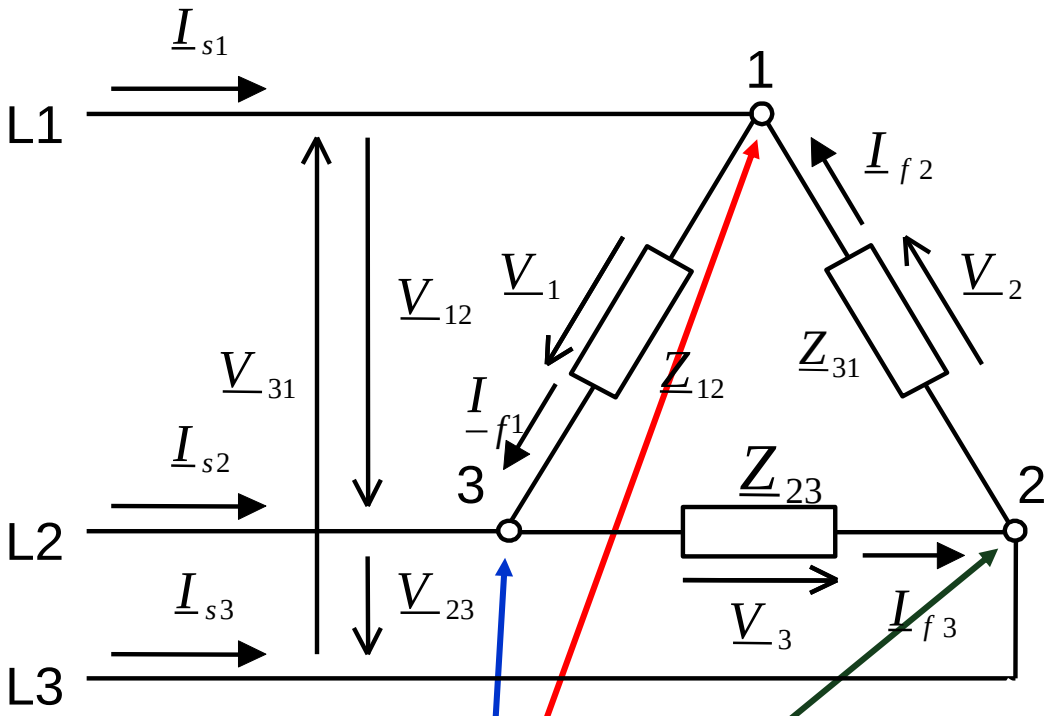
$$P = P_1 + P_2 + P_3 = V_{f1} \cdot I_{f1} \cdot \cos\varphi_1 + V_{f2} \cdot I_{f2} \cdot \cos\varphi_2 + V_{f3} \cdot I_{f3} \cdot \cos\varphi_3$$

For the symmetrical load

$$P = 3 \cdot V_f \cdot I_f \cdot \cos\varphi = 3 \cdot \frac{V_S}{\sqrt{3}} \cdot I_S \cdot \cos\varphi = \sqrt{3} \cdot V_S \cdot I_S \cdot \cos\varphi$$

Delta connection

symmetrical system

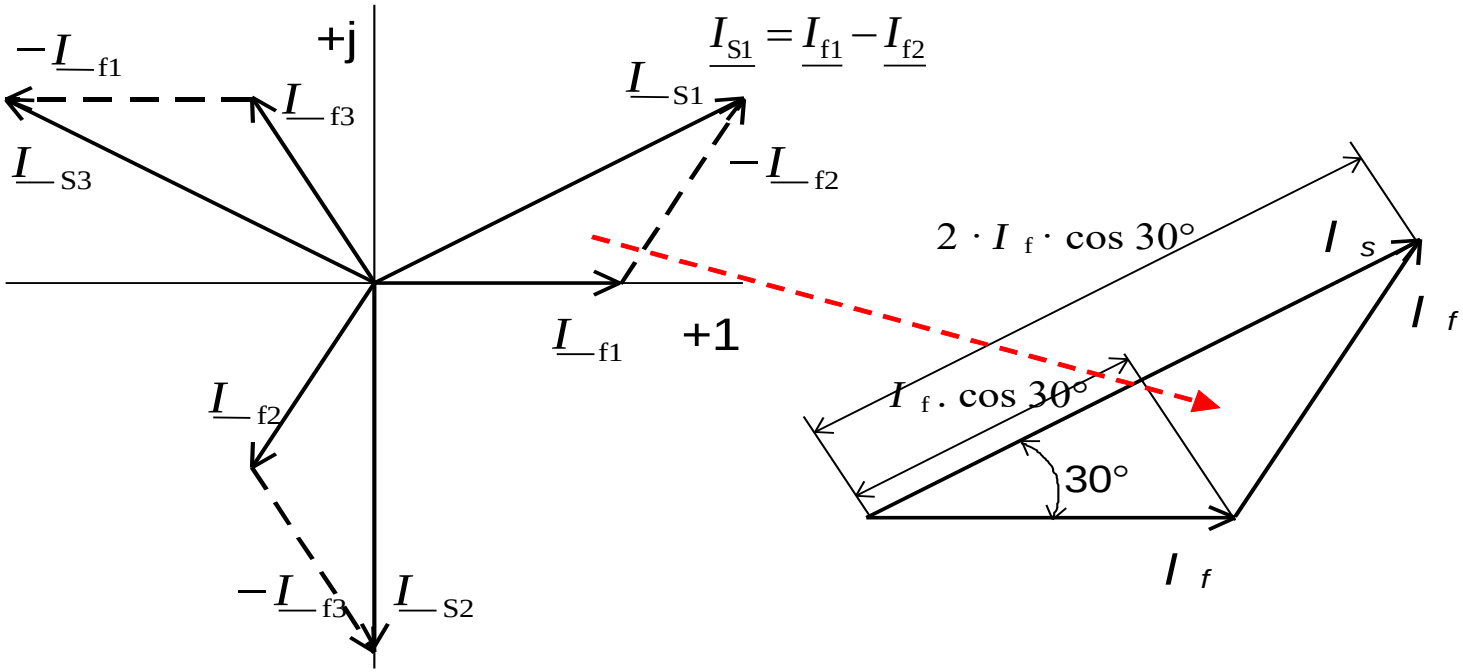


$$\underline{I}_{f1} - \underline{I}_{S1} - \underline{I}_{f2} = 0 \rightarrow \underline{I}_{S1} = \underline{I}_{f1} - \underline{I}_{f2}$$

$$\underline{I}_{f2} - \underline{I}_{S3} - \underline{I}_{f3} = 0 \rightarrow \underline{I}_{S2} = \underline{I}_{f2} - \underline{I}_{f3}$$

$$\underline{I}_{f3} - \underline{I}_{S2} - \underline{I}_{f1} = 0 \rightarrow \underline{I}_{S3} = \underline{I}_{f3} - \underline{I}_{f1}$$

Voltages and currents in the delta connection



$$I_{f1} = I_{f2} = I_{f3} = I_f$$

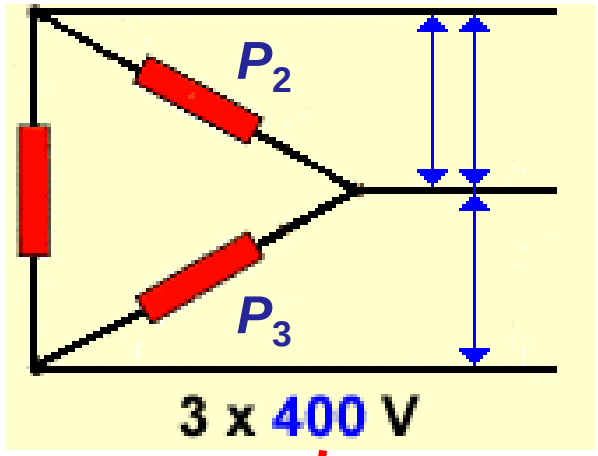
$$I_{S1} = I_{S2} = I_{S3} = I_S$$

$$I_S = 2 \cdot I_f \cdot \cos 30^0 \rightarrow I_S = \sqrt{3} \cdot I_f$$

$$U_S = U_f$$

Power in the delta connection

real power
in one phase



line voltage=phase voltage

real power of three phase load

$$P = P_1 + P_2 + P_3 = V_{f1} \cdot I_{f1} \cdot \cos\varphi_1 + V_{f2} \cdot I_{f2} \cdot \cos\varphi_2 + V_{f3} \cdot I_{f3} \cdot \cos\varphi_3$$

For the symmetrical load

$$P = 3 \cdot V_f \cdot I_f \cdot \cos\varphi = 3 \cdot V_S \frac{I_S}{\sqrt{3}} \cdot \cos\varphi = \sqrt{3} \cdot V_S \cdot I_S \cdot \cos\varphi$$

Non-symmetrical three-phase system

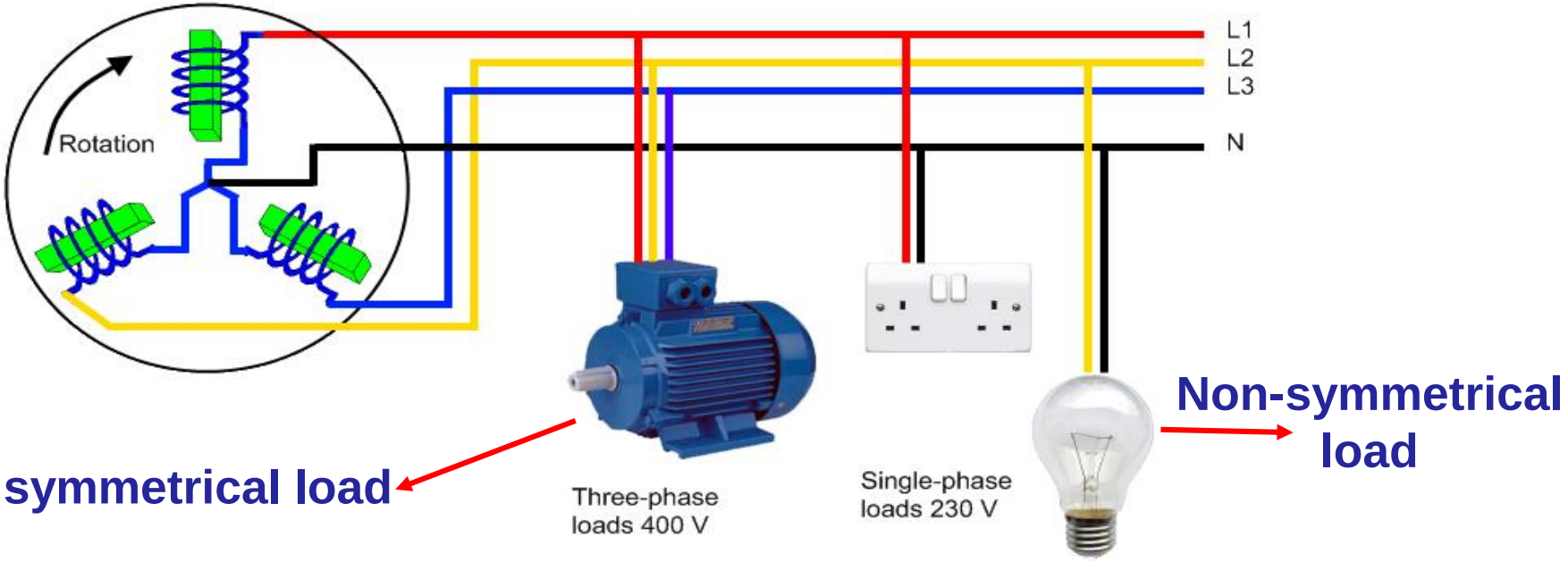
Non-symmetrical load

$$\underline{Z_1} \neq \underline{Z_2} \neq \underline{Z_3} \quad \left(Z_1 \neq Z_2 \neq Z_3 \neq Z, \varphi_1 \neq \varphi_2 \neq \varphi_3 \neq \varphi\right)$$

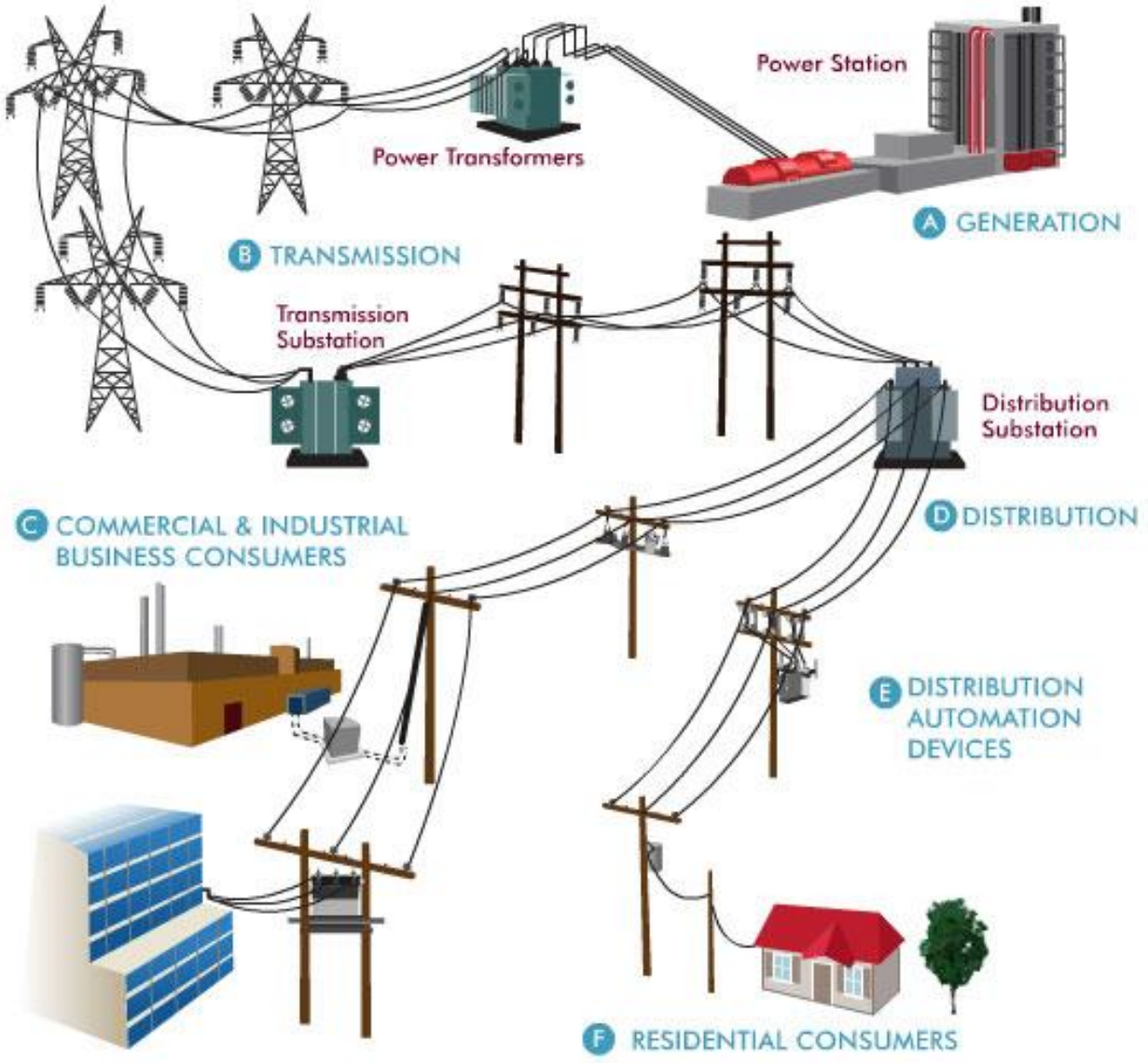
$$P = P_1 + P_2 + P_3 = V_{f1} \cdot I_{f1} \cdot \cos\varphi_1 + V_{f2} \cdot I_{f2} \cdot \cos\varphi_2 + V_{f3} \cdot I_{f3} \cdot \cos\varphi_3$$

$$S = S_1 + S_2 + S_3 = V_{f1} \cdot I_{f1} + V_{f2} \cdot I_{f2} + V_{f3} \cdot I_{f3}$$

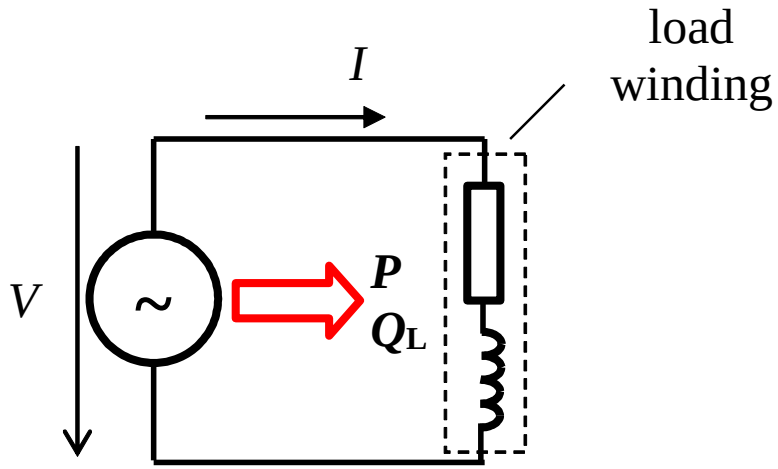
For star connection ⟹ $\underline{I_{f1}} + \underline{I_{f2}} + \underline{I_{f3}} = \underline{I_N} \neq 0$



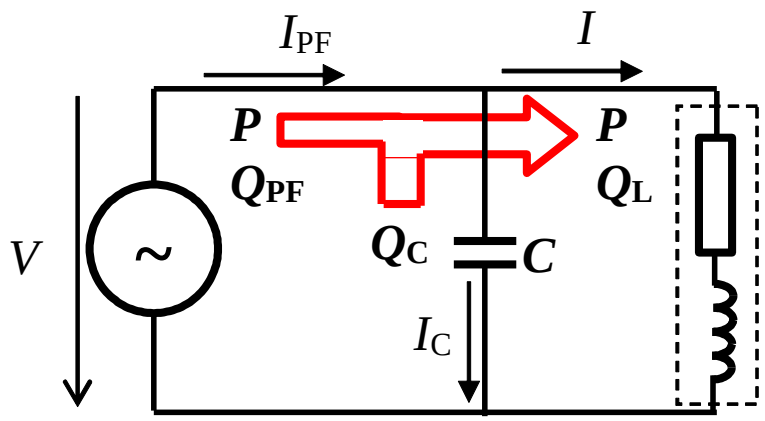
Electrical power network



Power factor correction

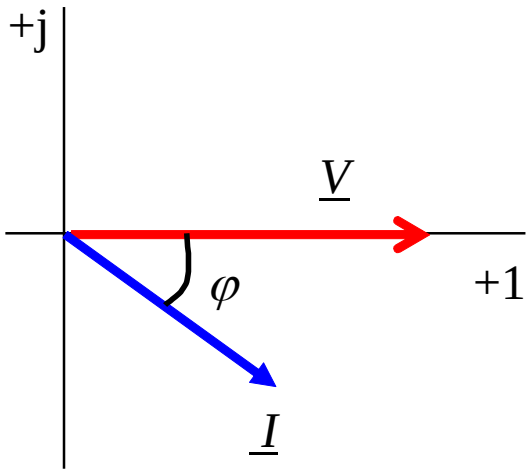


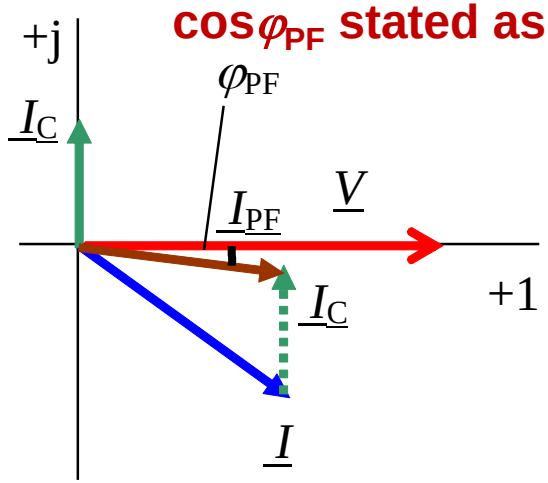
without power factor correction



with power factor correction

cosφ_{PF} stated as equal to 0,95





Reactive power of power-factor capacitor

$$Q_{\mathrm{L}} = Q_{\mathrm{PF}} + Q_{\mathrm{C}} \rightarrow Q_{\mathrm{C}} = Q_{\mathrm{L}} - Q_{\mathrm{PF}} = P \cdot \left(\mathrm{tg}\varphi - \mathrm{tg}\varphi_{\mathrm{PF}}\right)$$

Capacitance of power-factor capacitor

$$C = \frac{Q_{\mathrm{C}}}{V^2 \cdot \omega}$$

Benefits of power factor correction

- **reducing losses in power line**
- **increasing load carrying capabilities**

